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COMBINED RATIO-PRODUCT ESTIMATOR OF FINITE POPULATION MEAN IN STRATIFIED SAMPLING

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ABSTRACT

A procedure is given for estimating the population mean in stratified sampling in the presence of auxiliary information. The bias and variance of the proposed estimator have been derived under large sample approximation. Asymptotically optimum estimator in the class is identified with its variance formula. Estimator based on the estimated optimum is also investigated. It has been shown that the variance of the estimator based on estimated optimum value is same as that of optimum estimator. Both theoretical and empirical findings are encouraging and support the soundness of the proposed procedure for mean estimation.

Keywords: Finite population mean, Variance, Optimum estimator, Auxiliary variable, study variable.

Introduction

In planning surveys, Stratified sampling has often proved useful in improving the precision of other unstratified sampling strategies to estimate the finite population mean

$$\overline{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi}$$

with the notation used by *Dayal (1980)*. Other useful references for Stratified sampling are *Cochran (1977, chapter 5), Raj (1968, chapter 4), Rao (1962), Reddy (1978), Ruiz Espejo (1985, 1987), Som (1996, chapter 9 and 10)* and *Kadilar and Cingi (2003)*.

A ratio-product estimation of finite population mean $\overline{Y}$ can be made in two ways. One is to make a separate ratio-product estimate of the total of each stratum and add these totals. An alternative estimate is derived from a single combined ratio-product.

Consider a finite population of size N. Let *y* and *x* respectively, be the study and auxiliary variates on each unit U_j $(j = 1,2,3,...,N)$ of the population *U*. Let the population be divided into L strata with the h^{th}-stratum containing N_h units, *h=1,2,3,...,L* so that $\sum_{h=1}^{L} N_h = N$. Suppose that a simple random sample of size n_h is drawn without replacement (SRSWOR) from the h^{th}-stratum such that $\sum_{h=1}^{L} n_h = n$.

We compute the sample mean of the variates in stratified sampling method as

$$\overline{y}_{st} = \sum_{i=1}^{n_h} W_h \overline{y}_h \text{ and } \overline{x}_{st} = \sum_{i=1}^{n_h} W_h \overline{x}_h$$

where,

$W_h = \dfrac{N_h}{N}$ is stratum weight,

$\overline{y}_h$ is the sample mean of variate of interest in stratum *h* and

$\overline{x}_h$ is the sample mean of auxiliary variate in stratum *h*.

When the population mean $\overline{X}$ of the auxiliary variate *x* is known, *Hansen, Hurwitz, and Gurney (1946)* suggested a "combined ratio estimator"

$$\overline{y}_{Rc} = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right) \tag{1.1}$$

The "combined product estimator" for $\overline{Y}$ is defind by

$$\overline{y}_{Pc} = \overline{y}_{st}\left(\frac{\overline{x}_{st}}{\overline{X}}\right) \tag{1.2}$$

This estimator will often be used if the two variables are strongly positive correlated. To the first degree of approximation, the biases and variances of $\bar{y}_{Rc}$ and $\bar{y}_{Pc}$ are respectively given by

$$V(\bar{y}_{Rc}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{hy}^2 + R^2 S_{hx}^2 - 2RS_{hxy}\right) \tag{1.3}$$

$$V(\bar{y}_{Pc}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{hy}^2 + R^2 S_{hx}^2 + 2RS_{hxy}\right) \tag{1.4}$$

where, $C_{hx}^2 = \dfrac{S_{hx}^2}{\bar{X}_h^2}, C_{hy}^2 = \dfrac{S_{hy}^2}{\bar{Y}_h^2}, R = \dfrac{\bar{Y}}{\bar{X}}, K_h = \rho_{hxy}\dfrac{C_{hy}}{C_{hx}}, \rho_{hxy} = \dfrac{S_{hxy}}{S_{hx}S_{hy}}, \gamma_h = \left(\dfrac{1}{n_h} - \dfrac{1}{N_h}\right)$

$$S_{hy}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)^2,\ S_{hx}^2 = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(x_{hi} - \bar{X}_h\right)^2,\ S_{hxy} = \frac{1}{N_h - 1}\sum_{i=1}^{N_h}\left(y_{hi} - \bar{Y}_h\right)\left(x_{hi} - \bar{X}_h\right).$$

The linear regression estimator is more efficient than the ratio (product) estimator except when the regression line of y on x passes through the neighborhood of the origin in which case the efficiencies of these estimators are almost equal. However, owing to stronger intuitive appeal, survey statisticians favour the use of ratio (product) estimators. Further, we note that in many practical situations the regression line does not passes through the neighborhood of the origin. In these situations the ratio (product) estimator does not perform as well as the linear regression estimator. Considering this fact an attempt is made to improve the performance of suggested ratio-product estimator with their properties.

Proposed ratio-product estimator

We suggest the combined ratio-product estimator for estimating $\bar{Y}$ as

$$\hat{\bar{Y}}_{RP}^{(c)} = \bar{y}_{st}\left\{\alpha\frac{\bar{X}}{\bar{x}_{st}} + (1-\alpha)\frac{\bar{x}_{st}}{\bar{X}}\right\} \tag{2.1}$$

where α is a real constant to be determined such that the variance of $\hat{\bar{Y}}_{RP}^{(c)}$ is a minimum. For $\alpha = 1,\ \hat{\bar{Y}}_{RP}^{(c)} \rightarrow \bar{y}_{Rc}$ whereas for $\alpha = 0,\ \hat{\bar{Y}}_{RP}^{(c)} \rightarrow \bar{y}_{Pc}$.

To obtain the variance of $\hat{\bar{Y}}_{RP}^{(s)}$ to the first degree of approximation, we write

$$\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h = \bar{Y}(1 + e_0) \text{ and}$$

$$\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h = \bar{X}(1 + e_1)$$

such that,

$$E(e_0) = E(e_1) = 0,$$

under SRSWOR, we have

$$\left.\begin{aligned} E\left(e_0^2\right) &= \frac{1}{\overline{Y}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{hy}^2 \\ E\left(e_1^2\right) &= \frac{1}{\overline{X}^2}\sum_{h=1}^{L} W_h^2 \gamma_h S_{hx}^2 \\ E\left(e_0 e_1\right) &= \frac{1}{\overline{Y}\,\overline{X}}\sum_{h=1}^{L} W_h^2 \gamma_h S_{hyx} \end{aligned}\right\} \tag{2.2}$$

Expressing equation (2.1) in terms of e's we have

$$\hat{\overline{Y}}_{RP}^{(c)} = \overline{Y}\left(1+e_0\right)\left\{\alpha\left(1+e_1\right)^{-1} + \left(1+\alpha\right)\left(1+e_1\right)\right\}$$

We now assume that $|e_1| < 1$ so that we may expand $(1+e_1)^{-1}$ as a series in powers of e_1. Expanding multiplying out retaining terms of e's to the second degree, we obtain

$$\hat{\overline{Y}}_{RP}^{(c)} = \overline{Y}\left\{\alpha\left(1+e_0-e_1+e_1^2-e_0e_1\right) + \left(1-\alpha\right)\left(1+e_1+e_1+e_0e_1\right)\right\}$$

or

$$\hat{\overline{Y}}_{RP}^{(c)} - \overline{Y} = \overline{Y}\left\{e_0+e_1+e_0e_1+\alpha\left(e_1^2-2e_0e_1-2e_1\right)\right\} \tag{2.3}$$

Taking expectations of both sides of (2.3) and using the result (2.2), we obtain the bias of $\hat{\overline{Y}}_{RP}^{(c)}$ to order $o\left(n^{-1}\right)$ as

$$Bias\left(\hat{\overline{Y}}_{RP}^{(c)}\right) = \frac{1}{\overline{X}}\sum_{h=1}^{L} W_h^2 \gamma_h \left\{\alpha S_{hx}^2 + \left(1-2\alpha\right)S_{hyx}\right\} \tag{2.4}$$

Squaring both sides of equation (2.3) and again retaining terms to the second degree, we have

$$\left(\hat{\overline{Y}}_{RP}^{(c)} - \overline{Y}\right)^2 = \overline{Y}^2\left[e_0^2 + \left(1-2\alpha\right)\left\{\left(1-2\alpha\right)e_1^2 + 2e_0e_1\right\}\right] \tag{2.5}$$

Then the variance of $\hat{\overline{Y}}_{RP}^{(c)}$ to the first degree of approximation given as

$$V\left(\hat{\overline{Y}}_{RP}^{(c)}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{hy}^2 + \left(1-2\alpha\right)R\left\{\left(1-2\alpha\right)RS_{hx}^2 + 2S_{hxy}\right\}\right]$$

$$= \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{hy}^2 + \left(1-2\alpha\right)R^2 S_{hx}^2\left(1-2\alpha+2C^*\right)\right] \tag{2.6}$$

$$= V\left(\overline{y}_{st}\right) + R^2\left(1-2\alpha\right)V\left(\overline{x}_{st}\right)\left[\left(1-2\alpha\right)+2C^*\right] \tag{2.7}$$

where

$$C^* = \frac{Cov(\bar{y}_{st}, \bar{x}_{st})}{R\,V(\bar{x}_{st})} = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h S_{hxy}}{R\sum_{h=1}^{L} W_h^2 \gamma_h S_{hx}^2} = \frac{\beta^*}{R} = \frac{Cov(\bar{y}_{st}, \bar{x}_{st})}{R\sqrt{V(\bar{y}_{st})V(\bar{x}_{st})}}\sqrt{\frac{V(\bar{y}_{st})}{V(\bar{x}_{st})}} = \frac{\rho^*}{R}\sqrt{\frac{V(\bar{y}_{st})}{V(\bar{x}_{st})}} \qquad (2.8)$$

with $\beta^* = \dfrac{Cov(\bar{y}_{st}, \bar{x}_{st})}{V(\bar{x}_{st})}$ and $\rho^* = \dfrac{Cov(\bar{y}_{st}, \bar{x}_{st})}{R\sqrt{V(\bar{y}_{st})V(\bar{x}_{st})}}$

Minimization of (2.6) with respect to α we get the optimum value of α as

$$\alpha = \frac{1}{2}\left(1 + C^*\right) = \alpha_o \text{ (say)} \qquad (2.9)$$

Substitution of α_o in place of α in (2.1) yields the "optimum" estimator of $\bar{Y}$ as

$$\hat{\bar{Y}}_{RPO}^{(c)} = \frac{\bar{y}_{st}}{2}\left\{\frac{\left(1 + C^*\right)\bar{X}}{\bar{x}_{st}} + \frac{\left(1 - C^*\right)\bar{x}_{st}}{\bar{X}}\right\} \qquad (2.10)$$

Putting $\alpha = \alpha_o$ in (2.6) we get the variance of the optimum estimator $\hat{\bar{Y}}_{RPO}^{(c)}$ or the minimum variance of $\hat{\bar{Y}}_{RP}^{(c)}$ as

$$V\left(\hat{\bar{Y}}_{RPO}^{(c)}\right) = V(\bar{y}_{st})\left(1 - \rho^{*2}\right) \qquad (2.11)$$

$$= \sum W_h^2 \gamma_h \left(1 - \rho^{*2}\right) S_{hy}^2 = \min V\left(\hat{\bar{Y}}_{RP}^{(c)}\right) \qquad (2.12)$$

Thus we established the following theorem.

Theorem 2.1- To the first degree of approximation,

$$V\left(\hat{\bar{Y}}_{RPO}^{(c)}\right) \geq V(\bar{y}_{st})\left(1 - \rho^{*2}\right)$$

$$= \sum W_h^2 \gamma_h \left(1 - \rho^{*2}\right) S_{hy}^2 \qquad (2.13)$$

with equality if $\alpha = \dfrac{\left(1 + C^*\right)}{2}$

Remark 2.1 - The asymptotically optimum estimator (AOE) $\hat{\bar{Y}}_{RPO}^{(c)}$ can be used in practice only when the optimum value α_o of α is exactly known. But in practice it is hard to guess the value α_o exactly. However in repeated surveys or studies based on multiphase sampling, where information is gathered on several occasions (or based on past experience) it may be possible to guess the value of α_o (i.e. C^*) quite accurately. Even though this approach may reduce the precision, it may be satisfactory provided the relative decrease in precision is marginal, see *Sukhatme et. al. (1984, pp. 260).*

3. Allowable Departure from Optimum

Suppose $C_o^* = C(1-\eta)$; then

$$\alpha = \frac{1}{2}\left(1 + C^*\right) = \frac{1}{2}\{1 + C + C\eta\} = \frac{1+C}{2} + \frac{C\eta}{2} = \alpha_o + \frac{C\eta}{2} \tag{3.1}$$

Substituting this value of α in (2.6) we get the variance of $\hat{\bar{Y}}_{RP}^{(c)}$ as

$$V\left(\hat{\bar{Y}}_{RP}^{(c)}\right) = V(\bar{y}_{st})\left(1-\rho^{*2}\right)\left(1+\delta^{*2}\right) \tag{3.2}$$

$$= V\left(\hat{\bar{Y}}_{RPO}^{(c)}\right)\left(1+\delta^{*2}\right)$$

where $\delta^* = \dfrac{\rho^{*2}}{\left(1-\rho^{*2}\right)}\eta^2$

Now we have

$$\frac{V\left(\hat{\bar{Y}}_{RP}^{(c)}\right) - V\left(\hat{\bar{Y}}_{RPO}^{(c)}\right)}{V\left(\hat{\bar{Y}}_{RPO}^{(c)}\right)} = \delta^* = \frac{\rho^{*2}}{\left(1-\rho^{*2}\right)}\eta^2 \tag{3.3}$$

which follows that the proportional increase in variance of $\hat{\bar{Y}}_{RP}^{(c)}$ over that of AOE $\hat{\bar{Y}}_{RPO}^{(c)}$ is less than γ if

$$\frac{\rho^{*2}}{\left(1-\rho^{*2}\right)}\eta^2 < \gamma \quad \text{or} \quad |\eta| < \sqrt{\frac{\left(1-\rho^{*2}\right)}{\rho^{*2}}\gamma} \tag{3.4}$$

It follows from (3.4) that to ensure only a small relative increase in the variance, $|\eta|$ must be in the neighborhood of 'zero' if ρ^* is large but can depart considerably from 'zero' if ρ^* is moderate.

Comparisons

It is well known under stratified random sampling that

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{hy}^2 \tag{4.1}$$

From (2.7) and (4.1) we have

$$V(\bar{y}_{st}) - V\left(\hat{\bar{Y}}_{RP}^{(c)}\right) = (1-2\alpha)R\,V(\bar{x}_{st})\left[(1-2\alpha) + 2C^*\right]$$

which is possible if

$$\left.\begin{array}{ll} \textit{either} & \dfrac{1}{2} < \alpha < \dfrac{1}{2} + C^* \\ \textit{or} & \dfrac{1}{2} + C^* < \alpha < \dfrac{1}{2} \end{array}\right\} \tag{4.2}$$

Thus from (4.2) we note that $\hat{\bar{Y}}_{RP}^{(c)}$ is more efficient than $\bar{y}_{st}$ if

$$\min\left(\frac{1}{2}, \frac{1}{2} + C^*\right) < \alpha < \max\left(\frac{1}{2}, \frac{1}{2} + C^*\right) \tag{4.3}$$

From (2.7) and (1.3) we have

$$V\left(\bar{y}_{Rc}\right)-V\left(\hat{\bar{Y}}_{RP}^{(c)}\right)=4(1-\alpha)R^2\,V\left(\bar{x}_{st}\right)\left[\alpha-C^*\right]$$

which is possible if

$$\left.\begin{array}{ll} either & 1<\alpha<C^* \\ or & C^*<\alpha<1 \end{array}\right\} \qquad (4.4)$$

Thus from (4.4) we note that $\hat{\bar{Y}}_{RP}^{(c)}$ is more efficient than $\bar{y}_{Rc}$ if

$$\min\left(C^*,\,1\right)<\alpha<\max\left(C^*,\,1\right) \qquad (4.5)$$

From (2.7) and (1.4) we have

$$V\left(\bar{y}_{Pc}\right)-V\left(\hat{\bar{Y}}_{RP}^{(c)}\right)=4\alpha R^2 V\left(\bar{x}_{st}\right)\left[(1-\alpha)+2C^*\right]$$

which is possible if

$$\left.\begin{array}{ll} either & 0<\alpha<1+C^* \\ or & 1+C^*<\alpha<0 \end{array}\right\} \qquad (4.6)$$

Thus from (4.6) we note that $\hat{\bar{Y}}_{RP}^{(c)}$ is more efficient than $\bar{y}_{Pc}$ if

$$\min\left(0,\,1+C^*\right)<\alpha<\max\left(0,\,1+C^*\right) \qquad (4.7)$$

Estimator Based on Estimated 'Optimum'

If the experimenter is not able to specify the value precisely, then it may be desirable to estimate the optimum value $\alpha_o=\frac{\left(1+\hat{C}^*\right)}{2}$ from the sample yielding the combined estimator of $\bar{Y}$, defined below

$$\hat{\hat{\bar{Y}}}_{RPO}^{(c)}=\frac{\bar{y}_{st}}{2}\left\{\frac{\left(1+\hat{C}^*\right)\bar{X}}{\bar{x}_{st}}+\frac{\left(1-\hat{C}^*\right)\bar{x}_{st}}{\bar{X}}\right\} \qquad (5.1)$$

where $\hat{C}^*=\frac{\hat{\beta}^*}{\hat{R}}$, $\hat{\beta}^*=\frac{\sum_{h=1}^{L}W_h^2\gamma_h s_{hxy}}{\sum_{h=1}^{L}W_h^2\gamma_h s_{hx}^2}$, $\hat{R}=\frac{\bar{X}}{\bar{y}_{st}}$ (as $\bar{X}$ is known)

$$s_{hxy}=\frac{1}{\left(n_h-1\right)}\sum_{i=1}^{n_h}\left(x_{hi}-\bar{x}_h\right)\left(y_{hi}-\bar{y}_h\right) \text{ and } s_{hx}^2=\frac{1}{\left(n_h-1\right)}\sum_{i=1}^{n_h}\left(x_{hi}-\bar{x}_h\right)^2$$

If the allocation is proportional and $\frac{n_h}{n_h-1}\approx 1$, $\hat{\beta}^*$ reduces to the pooled estimator of the regression coefficient

$$\hat{\beta}^*=\frac{\sum_{h=1}^{L}\sum_{j=1}^{n_h}\left(y_{hj}-\bar{y}_h\right)\left(x_{hj}-\bar{x}_h\right)}{\sum_{h=1}^{L}\sum_{j=1}^{n_h}\left(x_{hj}-\bar{x}_h\right)^2}.$$

In order to obtained the variance of $\hat{\hat{\bar{Y}}}_{RPO}^{(c)}$, we write

$$\hat{C}^* = C^*(1+e_2)$$

with $E(\hat{C}^*) = C^* + O(n^{-1})$. Expressing $\hat{\hat{\bar{Y}}}_{RPO}^{(c)}$ in terms of e's we have

$$\hat{\hat{\bar{Y}}}_{RPO}^{(c)} = \frac{\bar{Y}}{2}(1+e_0)\left[\{1+C^*(1+e_2)\}(1+e_1)^{-1} + \{1-C^*(1+e_2)\}(1+e_1)\right]$$

where e_0 and e_1 are same as defined in section 2. Thus the variance of $\hat{\hat{\bar{Y}}}_{RPO}^{(c)}$ upto first degree of approximation is given as

$$V\left(\hat{\hat{\bar{Y}}}_{RPO}^{(c)}\right) = V(\bar{y}_{st}) - \frac{\{Cov(\bar{y}_{st}, \bar{x}_{st})\}^2}{V(\bar{x}_{st})} = V(\bar{y}_{st})(1-\rho^{*2})$$

(5.3)

which is same as that of $\hat{\bar{Y}}_{RP}^{(c)}$ i. e.

$$V\left(\hat{\hat{\bar{Y}}}_{RPO}^{(c)}\right) = V\left(\hat{\bar{Y}}_{RP}^{(c)}\right) \quad (5.4)$$

Empirical Study

To illustrate the performance of different estimators $\bar{y}_{st}, \bar{y}_{Rc}$ and $\hat{\bar{Y}}_{RP}^{(c)}$ over $\bar{y}_{st}$, we have considered the natural data given in *Singh and Chaudhary (1986, p.162)*.

The data were collected in a pilot survey for estimating the extent of cultivation and production of fresh fruits in three districts of Uttar- Pradesh in the year 1976-1977.

Stratum Number (h)	Total No. of village (N_h)	Total area (in hect.) (X_h)	No.of villages in sample (n_h)	Area under archards in ha. (x_h)	Total No. of trees (y_h)
1	985	11253	6	10.63	747
				9.90	719
				1.45	78
				3.38	201
				5.17	311
				10.35	448
2	2196	25115	8	14.66	580
				2.61	103
				4.35	316
				9.87	739
				2.42	196
				5.60	235

				4.70	212
				36.75	1646
3	1020	18870	11	11.60	488
				5.29	277
				7.94	374
				7.29	491
				8.00	499
				1.20	50
				11.50	455
				7.96	47
				23.15	879
				1.70	115
				2.01	115

The calculation have been shown in the given below

Stratum	W_h	γ_h	S_{hx}^2	S_{hy}^2	S_{hxy}
1	0.2345	0.16598	15.97	74775.47	1007.75
2	0.5227	0.12454	132.66	259113.71	5709.16
3	0.2428	0.08902	38.44	65885.60	1404.71

$R = 49.03$ and $\alpha_{opt} = 0.9422$.

The percent relative efficiency of an estimator *t* with respect to usual unbiased estimator $\bar{y}_{st}$ is defined by

$$PRE(t, \bar{y}_{st}) = \frac{V(\bar{y}_{st})}{V(t)} \times 100 \qquad (5.2)$$

We have computed the percent relative efficiency of $\bar{y}_{st}$, $\bar{y}_{Rc}$, $\bar{y}_{Pc}$ and $\hat{\bar{Y}}_{RP}^{(c)}$ with respect to $\bar{y}_{st}$ and presented in Table 5.1

Table 5.1
Showing the percent relative efficiencies of the various estimators of population mean $\bar{Y}$ with respect to stratified random sample mean $\bar{y}_{st}$

value of α	0.4	$0.5 = \bar{y}_{st}$	0.6	0.7

$PRE\left(\hat{\bar{Y}}_{RP}^{(c)}, \bar{y}_{st}\right)$	68.14	100.00	159.36	285.74
value of α	0.8	0.9	$\alpha_{opt} = 0.9422$	$1.0 = \bar{y}_{RC}$
$PRE\left(\hat{\bar{Y}}_{RP}^{(c)}, \bar{y}_{st}\right)$	597.28	1252.23	1400.55	1145.92
value of α	1.1	1.2	1.3	1.4
$PRE\left(\hat{\bar{Y}}_{RP}^{(c)}, \bar{y}_{st}\right)$	527.28	258.39	147.20	93.75

Table 5.2

Range of α for which $\hat{\bar{Y}}_{RP}^{(c)}$ is better than $\bar{y}_{st}$ and $\bar{y}_{Rc}$

$\hat{\bar{Y}}_{RP}^{(c)}$ is better than	
$\bar{y}_{st}$ if	$\bar{y}_{Rc}$ if
$\alpha \in (0.5, 1.3844)$	$\alpha \in (0.8844, 1)$

Conclusion

Table 5.1 and 5.2 clearly indicate that the proposed estimator $\hat{\bar{Y}}_{RP}^{(c)}$ is better than usual unbiased estimator $\bar{y}_{st}$ and combined ratio estimator in stratified sampling $\bar{y}_{Rc}$ even when α depends from its optimum value $\alpha_{opt} = 0.9422$. The range of α in which $\hat{\bar{Y}}_{RP}^{(c)}$ is better than $\bar{y}_{st}$ and $\bar{y}_{Rc}$ are $(0.5, 1.3844)$ and $(0.8844, 1)$. It is clear that the estimator $\hat{\bar{Y}}_{RP}^{(c)}$ is better than $\bar{y}_{st}$ for a wider range α while it is better than $\bar{y}_{Rc}$ for smaller range of α in which $\hat{\bar{Y}}_{RP}^{(c)}$ is better than $\bar{y}_{st}$ and $\bar{y}_{Rc}$ is $(0.8844, 1)$ which is short in length. The largest gain in efficiency by using $\hat{\bar{Y}}_{RP}^{(c)}$ over $\bar{y}_{st}$ and $\bar{y}_{Rc}$ is obtained when α attain its optimum value. The optimum estimator $\hat{\bar{Y}}_{RPO}^{(c)}$ (or the estimator $\hat{\hat{\bar{Y}}}_{RPO}^{(c)}$) is more efficient than $\bar{y}_{st}$ and $\bar{y}_{Rc}$ with substantial gain in efficiency. Thus the estimator $\hat{\hat{\bar{Y}}}_{RPO}^{(c)}$ is to be preferred in practice.

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